Pearson Edexcel

# Mark Scheme (Results)

## Summer 2022

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**PEARSON EDEXCEL IAL MATHEMATICS**

**General Instructions for Marking**

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

‘M’ marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

‘A’ marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

‘B’ marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Pure Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

**Method mark for solving 3 term quadratic:**

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question | Scheme | Marks |
|---|---|---|
| 1(a) | $A=\frac{1}{9}$ | B1 |
| | | (1) |
| (b) | $"3^{-2}"(1+\underline{(-2)(\frac{kx}{3})}+\underline{\frac{(-2)(-3)}{2}(\frac{kx}{3})^2}+...)$ or<br>$x:\quad 3^{-2}(-2)\left(\frac{k}{3}\right)\left(=-\frac{2k}{27}\right)$ and $x^2:\quad 3^{-2}\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2=\frac{k^2}{27}$ | B1 |
| | $\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2=3\times(-2)\left(\frac{k}{3}\right)\Rightarrow ...k^2=...k$ | M1 |
| | $k^2+6k=0$ * | A1* |
| | | (3) |
| (c)(i) | $k=-6$ | B1 |
| (ii) | $3^{-2}\frac{(-2)(-3)(-4)}{3!}\left(\frac{"-6"}{3}\right)^3=\frac{32}{9}$ | M1A1 |
| | | (3) |
| | | (7 marks) |

**Notes**

**Mark parts (a) and (b) as a whole.**

**(a)**

**B1:** $A=\frac{1}{9}$

**(b)**

**B1:** Correct unsimplified coefficients for $x$ and $x^2$ either in an expansion or separate for $(3+kx)^{-2}$ or for $\left(1+\frac{k}{3}\right)^{-2}$ (accept the $3^{-2}$ missing or incorrect). May be implied. Accept $B=-\frac{2k}{3}$ and $C=\frac{k^2}{3}$ if they forget the multiple outside. B0 if brackets on $\left(\frac{k}{3}\right)^2$ missing unless implied by recovery.

**M1:** Sets their coefficient of $x^2$ equal to 3 times their coefficient of $x$ to produce a two term quadratic equation in terms of $k$.

**A1*:** Achieves given answer from a correct equation, but condone if $B$ and $C$ both missed the $3^{-2}$. May be scored if $A$ was incorrect.

**(c)(i)**

**B1:** $k=-6$ only. The $k=0$ solution must be rejected.

**(ii)**

**M1:** Substitutes their non-zero value for $k$ into a correct expression for the coefficient of $x^3$. Must include the $3^{-2}$

**A1:** $\frac{32}{9}$ oe

| Question | Scheme | Marks |
|---|---|---|
| 2(a) | $\left(\dfrac{1}{(1+3x)(1-x)}=\dfrac{A}{1+3x}+\dfrac{B}{1-x}\Rightarrow\right)1=A(1-x)+B(1+3x)$ | B1 |
| | when $x=1 \Rightarrow 1=4B\Rightarrow B=...$ or when $x=-\dfrac{1}{3} \Rightarrow 1=\dfrac{4}{3}A\Rightarrow A=...$ | M1 |
| | $\dfrac{3}{4(1+3x)}+\dfrac{1}{4(1-x)}$ | A1 |
| | | **(3)** |
| (b) | $\int \cot y \, \mathrm{d}y=\int ... \, \mathrm{d}x \Rightarrow "\ln\sin y"=\int ... \, \mathrm{d}x$ | M1 |
| | $...=\int\left("\dfrac{3}{4(1+3x)}"+"\dfrac{1}{4(1-x)}"\right)\mathrm{d}x=...\ln(1+3x)\pm...\ln(1-x)\ \ (+c)$ | M1 |
| | $\ln\sin y=\dfrac{1}{4}\ln(1+3x)-\dfrac{1}{4}\ln(1-x)\ \ (+c)$ oe | A1ft |
| | $\ln\sin\left(\dfrac{\pi}{2}\right)=\dfrac{1}{4}\ln\left(1+3\times\dfrac{1}{2}\right)-\dfrac{1}{4}\ln\left(1-\dfrac{1}{2}\right)+c\Rightarrow c=...\left(=-\dfrac{1}{4}\ln 5\right)$ | dM1 |
| | $k\ln\sin y=m\ln(...)\Rightarrow\sin^k y=...^m$ or $k\ln\sin y=...\Rightarrow\sin^k y=\exp(...)$ | M1 |
| | $\sin^4 y=\dfrac{1+3x}{5(1-x)}$ | A1 |
| | | **(6)** |
| | | **(9 marks)** |

**Notes**

**(a)**

**B1:** For a correct suitable identity without fractions, such as $1=A(1-x)+B(1+3x)$, seen or implied.

**M1:** Attempts to find one of the constants by either substitution or equating coefficients. May be implied by a correct value for *A* or *B* via cover up rule.

**A1:** $\dfrac{3}{4(1+3x)}+\dfrac{1}{4(1-x)}$ oe allow values for *A* and *B* to be stated following a correct partial fraction form, or if correct partial fractions see in (b).

**(b)**

**M1:** Attempts to separate variables to form $\cot y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{g}(x)$ (oe for cot *y*) and integrate $\cot y$. Accept any changed function for the attempt but must be attempting to integrate cot *y* (oe).

**M1:** Attempts to integrate their partial fractions from (a) so award for $\dfrac{...}{(1+3x)}\to...\ln(1+3x)$ or $...\ln(4+12x)$

and $\dfrac{...}{(1-x)}\to...\ln(1-x)$ or $...\ln(4-4x)$ oe

**A1ft:** Correct expression (any equivalent) (both sides). Follow through on their constants for the partial fractions. Condone the absence of the constant of integration.

**dM1:** Depends on second M, and must have attempted to integrate both sides. Uses the initial conditions in an equation with a constant of integration. May integrate between limits to achieve this. (Accept if a value for *c* cannot be reached from their equation.)

**M1:** Attempts to rearrange their equation by correctly using log work to reach the required form $\sin^n y=\mathrm{f}(x)$. Must have had *k*ln sin *y* = … (*k* may be 1). Not dependent - may be gained before finding the constant if ln *A* is used, and allow if the constant is missing.

**A1:** $\sin^4 y=\dfrac{1+3x}{5(1-x)}$ (oe in correct form)

| Question | Scheme | Marks |
|---|---|---|
| 3(a) | $\frac{dA}{dt} = -0.5$ | B1 |
| | $A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$ | B1 |
| | $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{"-0.5"}{"2\pi x"} \quad \left(= \frac{-1}{4\pi x}\right)$ | M1 |
| | $\frac{dx}{dt} = -0.011368...$ | A1cso |
| | | **(4)** |
| (b) | $V = \pi x^2(3x) = 3\pi x^3$ | B1 |
| | $\frac{dV}{dx} = 9\pi x^2$ | B1ft |
| | $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times "-\frac{1}{4\pi x}" \quad (= -2.25x)$ | M1 |
| | $\left(\frac{dV}{dt} =\right) -9 \Rightarrow$ (Rate of decrease =) 9 $\left(\text{mm}^3\ \text{s}^{-1}\right)$ | A1 |
| | | **(4)** |
| | | **(8 marks)** |

**Notes**

**(a)**

**B1:** $\frac{dA}{dt} = -0.5$ seen or implied from working

**B1:** $\frac{dA}{dx} = 2\pi x$ seen or implied from working. Must be in terms of *x*, but allow recovery if in terms of *r* and later work uses $r = 7$ to achieve a solution.

**M1:** Attempts to use an appropriate chain rule with their $\frac{dA}{dt}$ and $\frac{dA}{dx}$ e.g. $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = ...$

**A1:** awrt $-0.0114$ or $-\frac{1}{28\pi}$ cso (must have the negative sign)

**(b)**

**B1:** $V = \pi x^2(3x)$ or $V = 3\pi x^3$

**B1ft:** $\frac{dV}{dx} = 9\pi x^2$ or ft from their equation for *V* in one variable

**M1:** Their $\frac{dV}{dx} \times$ their $\frac{dx}{dt}$. Note the $\frac{dx}{dt}$ must be in terms of *x* or with $x = 4$ substituted first, M0 if they use their answer to (a).

**A1:** (Rate of decrease = ) 9 $\left(\text{mm}^3\ \text{s}^{-1}\right)$ (with or without the negative sign). May be scored following $\frac{dA}{dt} = 0.5$ in part (a)

| Question | Scheme | Marks |
|---|---|---|
| 4(a) | $16x^3 - 9kx^2y + 8y^3 = 875$ | |
| | $(8)y^3 \to (8\times)3y^2\dfrac{dy}{dx}$ | B1 |
| | $-9kx^2y \to ...kxy \pm ... - 9kx^2\dfrac{dy}{dx}$ | M1 |
| | $48x^2 - 18kxy - 9kx^2\dfrac{dy}{dx} + 24y^2\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx}(24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \dfrac{dy}{dx} = ...$ | M1 |
| | $\dfrac{dy}{dx} = \dfrac{6kxy - 16x^2}{8y^2 - 3kx^2}$ * | A1* |
| | | (4) |
| (b) | $\dfrac{dy}{dx} = 0, x = \dfrac{5}{2} \Rightarrow \dfrac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0$ or<br>$x = \dfrac{5}{2} \Rightarrow 16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2 y + 8y^3 = 875$ | M1 |
| | $15ky - 100 = 0$ or $250 - \dfrac{225}{4}ky + 8y^3 = 875$ | A1 |
| | E.g. $16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2\left(\dfrac{20}{3k}\right) + 8\left(\dfrac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = ... \left(= \dfrac{64}{27}\right) \Rightarrow k = ...$ | M1 |
| | $k = \dfrac{4}{3}$ | A1 |
| | | (4) |
| | | **(8 marks)** |

**Notes**

**(a)**

**B1:** For $y^3 \to 3y^2\frac{dy}{dx}$. Allow if seen in aside working without the 8.

**M1:** Correct attempt at implicit differentiation on the $-9kx^2y$. Look for $-9kx^2y \to ...kxy \pm ... - 9kx^2\frac{dy}{dx}$

**M1:** Collects both of their $\frac{dy}{dx}$ terms together, collects non $\frac{dy}{dx}$ terms the other side of the equation, factorises and divides to achieve $\frac{dy}{dx} = ...$ Must have two $\frac{dy}{dx}$ terms, one from the attempt at differentiating $-9kx^2y$ and one from the attempt at differentiating $y^3$, but condone if an extra $\frac{dy}{dx} = ...$ term has been included.

**A1*:** Achieves $\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ with no errors

**(b)**

**M1:** Uses the information to produce one equation in *k* and *y,* e.g. sets the $\frac{dy}{dx}$ equal to 0 and substitutes $x = \frac{5}{2}$, or substitutes $x = \frac{5}{2}$ into the given equation. Allow one slip substituting.

**A1:** A correct equation without fraction and with simplified coefficients, so $15ky - 100 = 0$ oe or $250 - \frac{225}{4}ky + 8y^3 = 875$ oe

**M1:** For a complete method to find *k* so solves the equations simultaneously to achieve a value for *k*. May find *y* first e.g substitutes their $k = \frac{20}{3y}$ into the original equation, solves to find *y* and substitutes this back into $k = \frac{20}{3y}$ to find *k* via $250 - 375 + 8y^3 = 875 \Rightarrow y = 5 \Rightarrow k = \frac{20}{3\times 5} = ...$

**A1** $k = \frac{4}{3}$

Alt:

If they do not substitute $x = \frac{5}{2}$ initially then score

**M1:** Uses numerator of $\frac{dy}{dx}$ equal to 0 to find *y* in terms of *x* and *k* and substitute into original equation (allowing one slip)

**A1:** Correct equation:

$$6kxy - 16x^2 = 0 \Rightarrow y = \frac{8x^2}{3kx} \Rightarrow 16x^3 - 9kx^2\left(\frac{8x^2}{3kx}\right) + 8\left(\frac{8x^2}{3kx}\right)^3 = 875 \text{ oe}$$

**M1:** Substitutes $x = \frac{5}{2}$ and solves to find *k*

**A1:** $k = \frac{4}{3}$

| Question | Scheme | Marks |
|---|---|---|
| 5(a) | $1=2\sin u \Rightarrow p=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ | B1 |
| | $x=2\sin u \Rightarrow \frac{dx}{du}=2\cos u$ oe | M1 |
| | $\int \frac{3x+2}{\left(4-x^2\right)^{\frac{3}{2}}}\,dx=\int \frac{6\sin u+2}{\left(4-4\sin^2 u\right)^{\frac{3}{2}}}\,2\cos u\,du=\int \frac{6\sin u+2}{\left(4\cos^2 u\right)^{\frac{3}{2}}}\,2\cos u\,du$ | M1 |
| | $=\int \frac{12\sin u}{8\cos^2 u}+\frac{2}{4\cos^2 u}\,du=\int_0^{\frac{\pi}{6}}\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)du$ * | A1* |
| | | **(4)** |
| (b) | $\int\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)du=\frac{3}{2}\sec u+\frac{1}{2}\tan u$ | M1A1 |
| | $\left[\frac{3}{2}\sec u+\frac{1}{2}\tan u\right]_0^{"\frac{\pi}{6}"}=\left(\frac{3}{2}\sec\left("\frac{\pi}{6}"\right)+\frac{1}{2}\tan\left("\frac{\pi}{6}"\right)\right)-\left(\frac{3}{2}\sec 0+\frac{1}{2}\tan 0\right)=...$ | M1 |
| | $=\sqrt{3}+\frac{\sqrt{3}}{6}-\frac{3}{2}=\frac{7\sqrt{3}}{6}-\frac{3}{2}\quad\left(=\frac{7\sqrt{3}-9}{6}\right)$ | A1 |
| | | **(4)** |
| | | **(8 marks)** |

**Notes**

**(a)**

**B1:** $p=\frac{\pi}{6}$ Allow if seen anywhere, even in (b). $p$ = 30 is B0.

**M1:** $x=2\sin u \Rightarrow \frac{dx}{du}=\pm...\cos u$ or any rearrangement of this equation.

**M1:** **Full** substitution from an integral in terms of $x$ to an integral in terms of $u$ and **uses** the identity $\sin^2 u+\cos^2 u=1$ in the denominator. Do not be concerned with the limits for this mark.

**A1*:** Achieves given answer include d$u$ (with their $p$) with no errors and at least one intermediate step with the fractional power simplified. Condone missing d$u$ in intermediate lines.

**(b)**

**M1:** $\int\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)du=...\sec u+...\tan u$

**A1:** $\frac{3}{2}\sec u+\frac{1}{2}\tan u$ ignore any constant $c$

**M1:** Depends on having one term of the correct form, attempts to substitute in their $p$ ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).

**A1:** $\frac{7\sqrt{3}}{6}-\frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if $p$ = 30º was used.

| Question | Scheme | Marks |
|---|---|---|
| 6(a) | $\overrightarrow{AB}=\begin{pmatrix}5-1\\3--4\\-2-3\end{pmatrix}=\begin{pmatrix}4\\7\\-5\end{pmatrix}=4\mathbf{i}+7\mathbf{j}-5\mathbf{k}$ | M1 |
| | e.g. $\mathbf{r}=\mathbf{i}-4\mathbf{j}+3\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$ | M1A1 |
| | | **(3)** |
| (b) | $\overrightarrow{AC}=\begin{pmatrix}3-1\\p--4\\-1-3\end{pmatrix}=\begin{pmatrix}2\\p+4\\-4\end{pmatrix}=2\mathbf{i}+(p+4)\mathbf{j}-4\mathbf{k}$ | M1 |
| | $\begin{pmatrix}2\\p+4\\-4\end{pmatrix}\bullet\begin{pmatrix}4\\7\\-5\end{pmatrix}=8+7p+28+20=0\Rightarrow p=-8$ | M1A1 |
| | | **(3)** |
| (c) | $\lvert AB\rvert=\sqrt{4^2+7^2+(-5)^2}=\sqrt{90}$ or $\lvert AC\rvert=\sqrt{2^2+(-4)^2+(-4)^2}=6$ | M1 |
| | Area $\frac{1}{2}\times"\sqrt{90}"\times"6"=9\sqrt{10}$ | dM1A1 |
| | | **(3)** |
| | | **(9 marks)** |

**Notes**

**Accept either vector form throughout but extra i, j k in column vectors will lose A mark in (a).**
**(a) This is now being marked MMA**

**M1:** Attempts to find $\overrightarrow{AB}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

**M1:** Attempts equation for the line, score for $\overrightarrow{OA}+\lambda\times$ their $\overrightarrow{AB}$ or $\overrightarrow{OB}+\lambda\times$ their $\overrightarrow{AB}$ No need for **r** = for this mark.

**A1:** Any correct equation. Must be **r** = … (*l* = .. is A0)

**(b)**

**M1:** Attempts to find $\overrightarrow{AC}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

**M1:** Takes scalar product of their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$ to form and solve a linear equation in *p*

**A1:** $p=-8$

**(c)**

**M1:** Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$ using their *p*

**dM1:** Attempts to find the exact area of the triangle *ABC*. It is dependent on the previous method mark. There most common method will be $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|$ as in scheme but other methods are possible. E.g.

$\cos\angle ABC=\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}\Rightarrow A=\frac{1}{2}\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\sin\angle ABC$. Such a method must be complete, including use of Pythagorean identity to find $\sin\angle ABC$. Other more advanced methods (such as cross products) are also possible. If you see something you feel is worthy of some credit but does not fit the scheme, send to Review.

**A1:** $9\sqrt{10}$

| Question | Scheme | Marks |
|---|---|---|
| 7(a) | $\frac{dx}{dt} = \cos t + 6\cos t \sin t \qquad \frac{dy}{dt} = 3\cos t - 2\sin t$ | B1B1 |
| | $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3\cos t - 2\sin t}{\cos t + 6\cos t \sin t} = \frac{3\cos\pi - 2\sin\pi}{\cos\pi + 6\cos\pi\sin\pi} = 3$ * | M1A1* |
| | | **(4)** |
| (b) | When $t = \pi,\ x = -3,\ y = -2$ | B1 |
| | $y - "-2" = 3(x - "-3")$ | M1 |
| | $y = 3x + 7$ | A1 |
| | | **(3)** |
| (c) | $y = 3x + 7 \Rightarrow 3\sin t + 2\cos t = 3\left(\sin t - 3\cos^2 t\right) + 7$ or<br>$y = 3\left(x + 3\cos^2 t\right) + 2\cos t \Rightarrow 3x + 7 = 3x + 9\cos^2 t + 2\cos t$ | M1 |
| | $\Rightarrow 9\cos^2 t + 2\cos t - 7 = 0$ * | A1* |
| | | **(2)** |
| (d) | $\cos t = \frac{7}{9}$ | B1 |
| | $y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$ | M1A1 |
| | | **(3)** |
| | | **(12 marks)** |

**Notes**

**(a)**

**B1**: $\left(\frac{dx}{dt} =\right) \cos t + 6\cos t \sin t$ or $\cos t + 3\sin 2t$

**B1**: $\left(\frac{dy}{dt} =\right) 3\cos t - 2\sin t$

**M1:** Attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t = \pi$. (May substitute $\pi$ before dividing.)

**A1*:** Achieves $\frac{dy}{dx} = 3$ with full working shown and no errors.

**(b)**

**B1:** $x = -3,\ y = -2$ which may be seen within their working

**M1:** Attempts to find the equation of the tangent with gradient 3. If they use $y = mx + c$ they must proceed as far as $c = ...$

**A1:** $y = 3x + 7$

**(c)**

**M1:** A full attempt to solve simultaneously the given parametric equations with their equation of the tangent

**A1*:** Achieves $9\cos^2 t + 2\cos t - 7 = 0$ with no errors

**(d)**

**B1**: $\cos t = \frac{7}{9}$ seen or implied. Allow if seen in (c).

**M1:** Attempts to find the $y$ coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for cos $t$

**A1:** $\frac{4\sqrt{2}}{3} + \frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.

| Question | Scheme | Marks |
|---|---|---|
| 8(a) | $V=\pi\int_0^{10}\left(10xe^{-\frac{1}{2}x}\right)^2 \mathrm{d}x =;\ 100\pi\int_0^{10} x^2\mathrm{e}^{-x}\ \mathrm{d}x$ | M1;A1 |
| | | (2) |
| (b) | $\int x^2\mathrm{e}^{-x}\ \mathrm{d}x = -x^2\mathrm{e}^{-x}+2\int x\mathrm{e}^{-x}\,\mathrm{d}x$ | M1 |
| | $=-x^2\mathrm{e}^{-x}+2\int x\mathrm{e}^{-x}\,\mathrm{d}x=-x^2\mathrm{e}^{-x}+2\left\{-x\mathrm{e}^{-x}+\int \mathrm{e}^{-x}\mathrm{d}x\right\}$ | dM1 |
| | $-x^2\mathrm{e}^{-x}-2x\mathrm{e}^{-x}+2\int \mathrm{e}^{-x}\mathrm{d}x=-x^2\mathrm{e}^{-x}-2x\mathrm{e}^{-x}-2\mathrm{e}^{-x}\ (+c)$ | A1 |
| | | (3) |
| (c) | Total volume $=2\times\text{"}100\pi\text{"}\int_0^{10} x^2\mathrm{e}^{-x}\,\mathrm{d}x$ | M1 |
| | $\int_0^{10} x^2\mathrm{e}^{-x}=\left[-x^2\mathrm{e}^{-x}-2x\mathrm{e}^{-x}-2\mathrm{e}^{-x}\right]_0^{10}=\left(-(10)^2\mathrm{e}^{-10}-2\times10\mathrm{e}^{-10}-2\mathrm{e}^{-10}\right)-\left(-2\right)$ | M1 |
| | $=2-122\mathrm{e}^{-10}\quad(1.9944\ldots)$ | A1 |
| | $\text{Density}=\dfrac{5000}{\text{"}200\pi\text{"}\times\text{"}1.9944...\text{"}}$ | dM1 |
| | awrt 3.99 (g / cm$^3$) | A1 |
| | | (5) |
| | | (10 marks) |

**Notes**

**(a)**

**M1:** Forms a correct unsimplified expression for the volume.

**A1:** Achieves $100\pi\int_0^{10} x^2\mathrm{e}^{-x}\ \mathrm{d}x$ Condone a missing d$x$ but limits must be present.

**(b)**

**M1:** Attempts integration by parts in the right direction to achieve an expression of the form $...x^2\mathrm{e}^{-x}\pm...\int x\mathrm{e}^{-x}\,\mathrm{d}x$ Condone missing d$x$

**dM1:** Dependent on the previous method mark. Attempts integration by parts a second time to achieve an expression of the form $...x^2\mathrm{e}^{-x}\pm...x\mathrm{e}^{-x}\pm...\int \mathrm{e}^{-x}\,\mathrm{d}x$ Condone missing d$x$

**A1:** $-x^2\mathrm{e}^{-x}-2x\mathrm{e}^{-x}-2\mathrm{e}^{-x}\ (+c)$ with or without the constant of integration

For attempts via the DI (tabular) method, look for first two rows of the table to have correct forms for M1, all rows with correct forms and answer extracted for dM1 and A1 for correct answer.

**(c)**

**M1:** A correct strategy to find the total volume with their values of $k$.

**M1:** Substitutes the limits of 10 and 0 into their part (b) and subtracts. Alternatively allow M1 for limits 20 and 0 used (as a mistaken attempt to double).

**A1:** $2-122\mathrm{e}^{-10}$ or awrt 1.99

**dM1:** Dependent on second M. Attempts to find the density using $\dfrac{5000}{\text{their Volume}}$. The attempt at the volume need not be correct but an attempt at using (b) must have been made. E.g. if they forget $k$ or forget to double, allow for the attempt with their volume. Must be with 5000 in numerator, or with correct work to reach correct units later.

**A1:** awrt 3.99 (g / cm$^3$) oe. Accept exact simplified answers such as $\dfrac{5000}{200\pi(2-122\mathrm{e}^{-10})}$

| Question | Scheme | Marks |
|---|---|---|
| 9 | For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader.<br>**M1:** Will be scored for setting up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage.<br>**A1:** Scored for a correct statement from which it is possible to draw a contradiction.<br>**dM1;** For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions.<br>**A1:** Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technicality a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume $n^2-2$ is divisible be 4 for all *n*" | |
| 9 | (Assume that there is an *n* with $n^2-2$ is divisible by 4 so) $n^2-2=4k$ | M1 |
| | then $n^2=4k+2=2(2k+1)$ (so is even) | A1 |
| | Hence $n^2$ is even so *n* (=2*m*) is even hence $n^2$ is a multiple of 4<br>As $n^2$ is a multiple of 4 then $n^2-2=4m^2-2=2(2m^2-1)$ cannot be a multiple of 4 (as $2m-1$ is odd) so there is a contradiction. | dM1 |
| | So the original assumption has been shown false.<br>Hence "$n^2-2$ is never divisible by 4" is true for all *n* * | A1* |
| | | **(4)** |
| | | **(4 marks)** |

**Notes**

**M1:** Sets up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$

**A1:** Reaches $n^2=2(2k+1)$

**dM1:** For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

Accept explanations such as "as $n^2$ is even then *n* is even hence $n^2$ is a multiple of 4 so $n^2-2$ cannot be a multiple of 4 (as 4 does not divide 2)"

**A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2-2$ is divisible be 4 for all *n*"

| | | |
|---|---|---|
| **9**<br>**Alt 1** | (Assume that $n^2-2$ is divisible by 4 for some *n*,) so $\frac{n^2-2}{4}$ is an integer. Then if *n* is even *n* = 2*m* (*m* integer) so $\frac{n^2-2}{4}=\frac{(2m)^2-2}{4}$ (oe with odd) | M1 |
| | $=m^2-\frac{1}{2}$ (which is not an integer) | A1 |

| | | |
|---|---|---|
| | Since $m^2$ is an integer, $m^2-\frac{1}{2}$ is not, hence $n$ cannot be even, but if $n$ is odd then $\frac{n^2-2}{4}=\frac{(2m+1)^2-2}{4}=m^2+m-\frac{1}{4}$, which is again not an integer (since $m^2+m$ is) | dM1 |
| | Hence there is a contradiction (as $n$ cannot be an integer)<br>Hence "$n^2-2$ is never divisible by 4" is true for all $n$ * | A1* |
| | | **(4)** |
| | | **(4 marks)** |

**Notes**

**M1:** Sets up an algebraic statement in terms of a variable (integer) $m$ or any other variable aside $n$ that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this Alt, consider case use of $n = 2m$ or $n = 2m + 1$ in $\frac{n^2-2}{4}$ is sufficient

**A1:** Reaches $m^2-\frac{1}{2}$ for $n$ even or $m^2+m-\frac{1}{4}$ for $n$ odd.

**dM1:** For a complete argument that leads to a contradiction in both cases. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

**A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all $n$. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2-2$ is divisible be 4 for all $n$"

| | | |
|---|---|---|
| **9 Alt 2** | (Assume that $n^2-2$ is divisible by 4) $\Rightarrow n^2-2=4k$ | M1 |
| | $\Rightarrow n^2=4k+2 \Rightarrow n=2\sqrt{k+\frac{1}{2}}$ or $n=\sqrt{2}\sqrt{2k+1}$ | A1 |
| | So for some integer $m$ $\sqrt{k+\frac{1}{2}}=\frac{\text{m}}{2} \Rightarrow 2k+1=\frac{m^2}{2}$ but $m^2$ is odd if $m$ is odd so $\frac{m^2}{2}$ not an integer, or $m^2$ is a multiple of 4 if $m$ even, so odd=even<br>or $2k + 1$ is odd, so does not have a factor 2 to combine with the $\sqrt{2}$ outside, hence $n$ must be irrational | dM1 |
| | Hence we have a contradiction.<br>So "$n^2-2$ is never divisible by 4" is true for all $n$ * | A1* |
| | | **(4)** |
| | | **(4 marks)** |

**Notes**

**M1:** Sets up an algebraic statement in terms of a variable (integer) $k$ or any other variable aside $n$ that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$

**A1:** Reaches $n=2\sqrt{k+\frac{1}{2}}$ or $n=\sqrt{2}\sqrt{2k+1}$

| | | |
|---|---|---|
| **dM1:** For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Must be a valid attempt to show that $2\sqrt{k+\frac{1}{2}}$ / $\sqrt{2}\sqrt{2k+1}$ is not an integer, and this method is a hard route.<br>**A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2-2$ is divisible be 4 for all *n*" | | |
| **9**<br>**Alt 3** | (Assume that $n^2-2$ is divisible by 4) then for *n* even we have (for some integer *m*) $n^2-2=4m^2-2$ **or** for *n* odd $n^2-2=4\left(m^2+m\right)-1$ | M1 |
| | $4m^2-2$ or $4\left(m^2+m\right)-1$ | A1 |
| | Since 4 divides $n^2-2$ and $4m^2$ thus for *n* even, 4 must divide 2, a contradiction, so *n* cannot be even, and also 4 divides $4\left(m^2+m\right)$ so for *n* odd, 4 divides 1, also a contradiction. | dM1 |
| | Hence we have a contradiction for both cases (and as *n* must be either even or odd). so "$n^2-2$ is never divisible by 4" is true for all *n* * | A1* |
| | | **(4)** |
| | | **(4 marks)** |

**Notes**

**M1:** Sets up an algebraic statement in terms of a variable (integer) *m* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing using *n* odd or *n* even to form an expression for $n^2-2$ of the form $4\times\text{integer}\pm\text{non-mulitple of 4}$

**A1:** Reaches $4m^2-2$ or $4\left(m^2+m\right)-1$

**dM1:** For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Both cases must be considered with a reason for the contradiction given (not just stated not divisible by 4).

**A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2-2$ is divisible be 4 for all *n*"

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